# MTH 306 J&K Lab 7
# The TD diagram for 2D autonomous linear systems

Friday, March 28, 2025

This lab should all be done with **pencil and paper and your brain**, *without consulting the internet*.
Turn in the completed worksheet to the TA.

The characteristic polynomial for the general 2x2 matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ is } det \begin{bmatrix} a-\lambda & b \\ c & d-\lambda \end{bmatrix}.$$

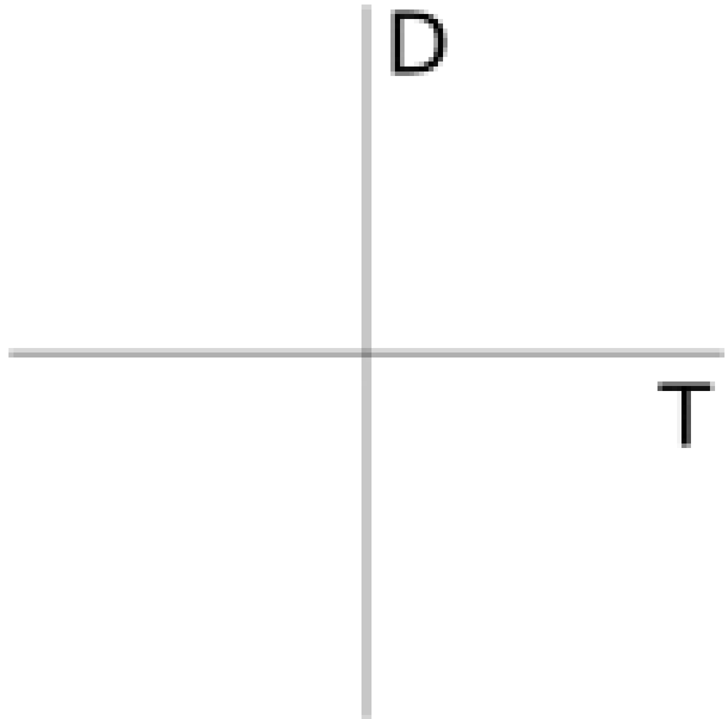
1. Expand it out, and rewrite it using the short-hand
$T$ for $a + d$ (the "trace" of $A$) and $D$ for $ad - bc$ (the determinant of $A$).

2. Use the quadratic formula to get an expression for the eigenvalues (roots of the characteristic polynomial) **-** in terms of T & D.

3. (a) What is the sum of the two eigenvalues (in terms of T and/or D)?

(b) What is their product (in terms of T and/or D)?

4. Under what conditions will we have complex eigenvalues?
In “TD diagram” below, qualitatively sketch the boundary of the region, and lightly shade the complex side.




5. Where in the TD diagram is the "Emily & Jacob" example? (See boards Day 15.) Mark it with a dot labeled “5”.

6. Find an example of a nodal sink in your class notes and mark the corresponding point in the TD diagram.

7. Which region of the TD diagram corresponding to nodal sinks?
Hint: how can you characterize two numbers being both negative in terms of their sum and their product?
Shade this region using a different color/pattern than you used in Question 4.

8. (a) Find an example of a saddle in your class notes and mark the corresponding point in the TD diagram.
(b) What condition on T and D indicates a saddle (one eigenvalue negative, one positive)?
Shade the region of saddles in the TD diagram.

9. Where are the nodal sources? Write “nodal sources” in the appropriate place in your diagram.

10. Indicate on your diagram where the spiral sinks and spiral sources are respectively.

11. Where in the diagram do the harmonic oscillators "live"? The harmonic oscillator equation is $\frac{d^2y}{dt^2}+2p\frac{dy}{dt}+q\,y=0$ with constants $p \geq 0$ and $q > 0$, which you'll have to convert to a first order system in order to answer this question.