

TD diagram (Lab 7, S2S)

TD diagram ①

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\begin{aligned} \textcircled{1} \det(A - \lambda I) &= \begin{vmatrix} a - \lambda & b \\ c & d - \lambda \end{vmatrix} = (a - \lambda)(d - \lambda) - bc \\ &= \lambda^2 - (a + d)\lambda + (ad - bc) \\ &= \lambda^2 - T\lambda + D \end{aligned}$$

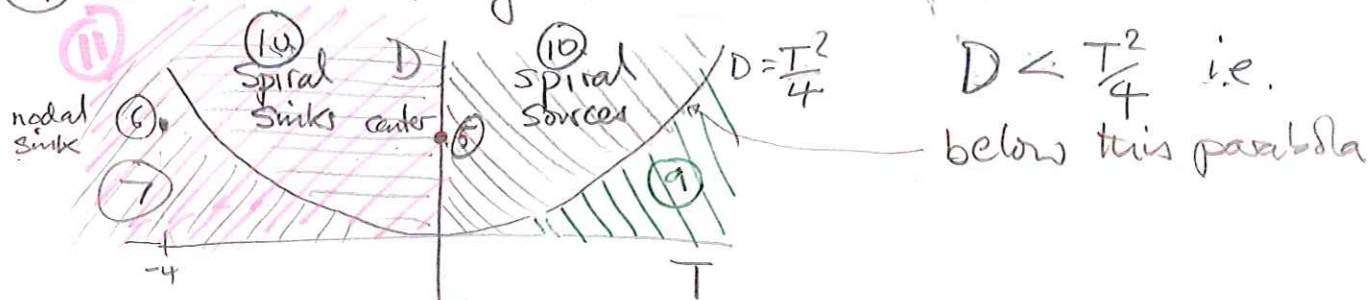
$$\begin{aligned} \textcircled{2} \lambda^2 - T\lambda + D &= 0 \\ \lambda &= \pm \frac{T}{2} \pm \frac{1}{2}\sqrt{T^2 - 4D} \end{aligned}$$

$$\textcircled{3} \lambda_+ + \lambda_- = \frac{T}{2} + \frac{1}{2}\sqrt{} + \frac{T}{2} - \frac{1}{2}\sqrt{} = T$$

$$\begin{aligned} \lambda_+ \lambda_- &= \left(\frac{T}{2} + \frac{1}{2}\sqrt{}\right)\left(\frac{T}{2} - \frac{1}{2}\sqrt{}\right) = \frac{T^2}{4} - \frac{1}{4}(T^2 - 4D) \\ &= D \end{aligned}$$

(cross terms cancel)

④ Distinct real eigenvalues when $\sqrt{T^2 - 4D} > 0$



⑤ Emory & Jacob (Day 15) $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ $T=0, D=1$

⑥ A nodal sink $A = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$ $T=-4, D = (-2)(-2) - (1)(1) = 4 - 1 = 3$

$D < \frac{T^2}{4} = \frac{16}{4} = 4$

⑦ Both λ 's negative $\Rightarrow D = \text{product} > 0$, $T = \text{sum} < 0$
and need $D < \frac{T^2}{4}$ for real.

⑧ A saddle $A = \begin{bmatrix} -3 & -5 \\ 0 & 2 \end{bmatrix}$ $T = -3 + 2 = -1, D = (-3)(2) = -6$

(9) Nodal sources.

2 positive $\Rightarrow T = \text{sum} > 0$, $D = \text{product} > 0$
 real $\rightarrow D < \frac{T^2}{4}$: below parabola.

(10) spiral sinks & sources

complex $\Rightarrow D > \frac{T^2}{4}$: above parabola

Sink: real part negative $\rightarrow$ sum negative $\Rightarrow T < 0$
 source: real part positive $\rightarrow$ sum positive $\Rightarrow T > 0$

(11) $y'' + 2py' + qy$, $p \geq 0$, $q > 0$

$$\begin{aligned} y' &= v \\ v' &= -qy - 2pv \end{aligned} \rightarrow \begin{bmatrix} y \\ v \end{bmatrix}' = \begin{bmatrix} 0 & 1 \\ -q & -2p \end{bmatrix}$$

$T = -2p \leq 0$
 $D = q > 0$ } so live in 2nd quadrant of TD